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Basic Frequency Modulation Synthesis technique

History

FM, short for Frequency Modulation, was invented in the late 1960s by the composer John Chowning. This technique was first explored because time-varing additive synthesis was costly at the time from a computational standpoint: synthesis was basically developed on fixed-waveform. FM synthesis is based on the same principles used for FM radio trasmission. Chowning's first experiments employed an audio signal (called a modulator) to control the frequency of an oscillator (called a carrier)

Simple FM

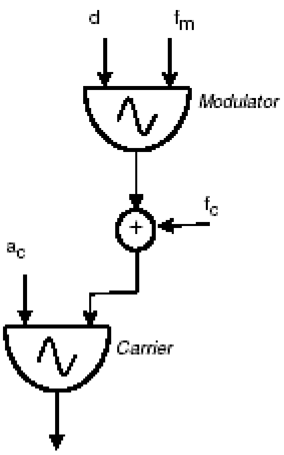
Simple FM is based upon two fundamental principles:

- the carrier
- the modulator.

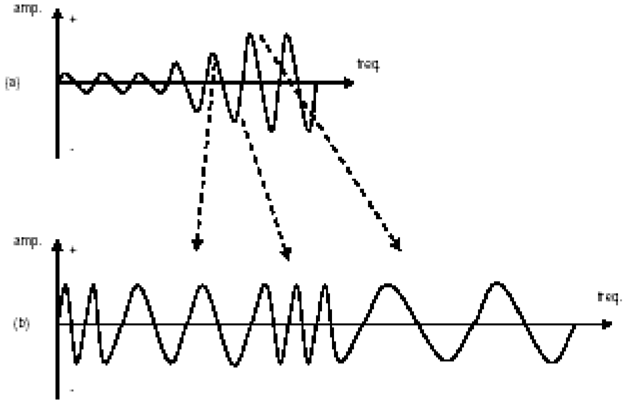
In the basic FM technique (referred to as simple FM), a carrier oscillator is modulated in frequency by a modulator oscillator. These are the parameters for a simple FM instrument:

- d = frequency deviation
- f_m = modulator frequency
- a_c = carrier amplitude
- f_c = offset carrier frequency.

The figure below illustrates a simple frequency modulation architecture.



The output of the modulator is offset by a constant, represented as f_c, and the result is then applied to control the frequency of the carrier. If the "amplitude" of the modulator is equal to zero, then there is no modulation. In this case, the output from the carrier will be a simple sinewave at frequency f_c. Conversely, if the "amplitude" of the modulator is greater than zero, then modulation occurs and the output from the carrier will be a signal whose frequency deviates proportionally to the "amplitude" of the modulator. There is a special name in FM theory for this particular word "amplitude": frequency deviation and its value is usually expressed in Hz. These parameters frequency deviation and modulator frequency can drastically change the form of the carrier wave. If the modulator frequency is kept constant whilst increasing the frequency deviation, then the period of the carrier's output will increasingly expand and contract, proportional to the frequency deviation. If the frequency deviation remains constant and the modulator frequency is increased, then the rate of the deviation will become faster.

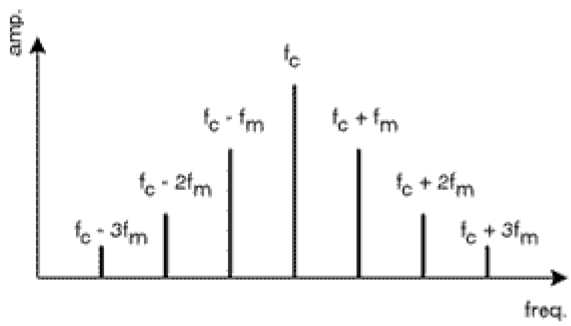


Spectrum of FM sound

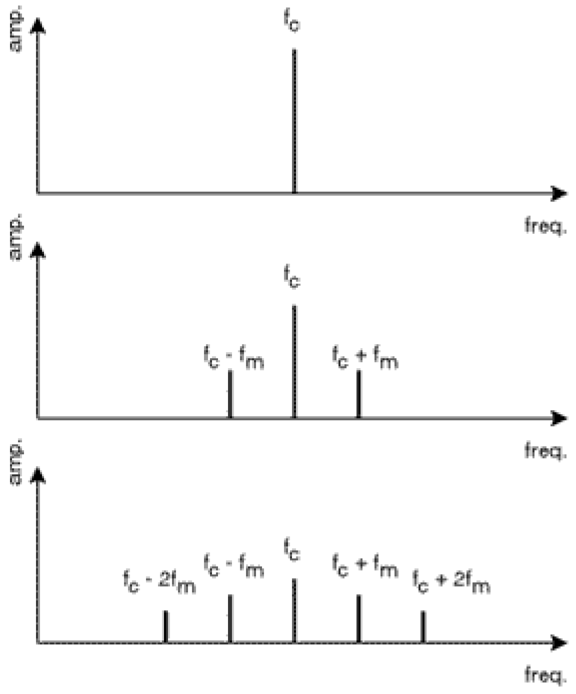
The simplicity of its architecture and its capability to produce a great variety of different timbres made FM synthesis more attractive than other techniques available at the time of its invention. Moreover, it does not necessarily need waveforms other than sinusoids, for both modulator and carrier, in order to produce interesting musical sounds.

Calculating the frequencies of the partials

The spectrum of an FM sound is composed of the offset carrier frequency (f_c) and a number of partials on either side of it, spaced at a distance equal to the modulator frequency (f_m). The partials generated on each side of the carrier frequency are usually called sidebands. The sideband pairs are calculated as follows: $f_c + k \times f_m$ and $f_c - k \times f_m$ where k is an integer, greater than zero, which corresponds to the order of the partial counting from f_c.



The amplitudes of partials are determined mostly by the frequency deviation. When there is no modulation (i.e. $d = 0$) the power of the signal resides entirely in the offset carrier frequency f_c. Increasing the value of d produces sidebands at the expense of the power in f_c. The greater the value of d, the greater the number of generated partials and, therefore, the wider the distribution of the power between the sidebands. The FM theory provides a useful tool for the control of the number of audible sideband components and their respective amplitudes: the modulation index, represented as i. The modulation index is the ratio between the frequency deviation and the modulator frequency: $i = d/f_m$. As the modulation index increases from zero, the number of audible partials also increases and the energy of the offset carrier frequency is distributed among them. The number of sideband pairs with significant amplitude can generally be predicted as $i + 1$; for example, if $i = 3$ than there will be four pairs of sidebands surrounding f_c.

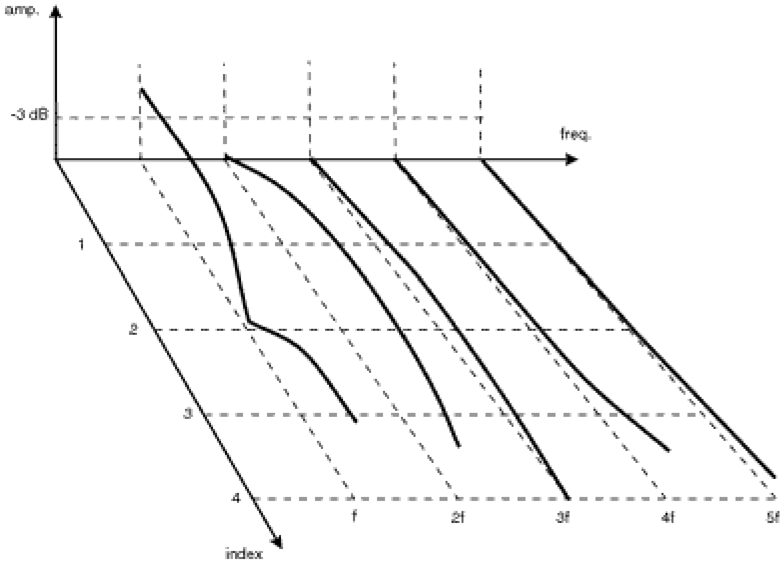


Calculating the amplitudes of the partials

The offset carrier frequency may often be the most prominent partial in an FM sound; in this case it will define the pitch of the sound. Sound engineers tend to refer to the carrier frequency as the fundamental frequency, but musicians often avoid this nomenclature because in music the term "fundamental frequency" is normally associated with pitch and in FM the carrier frequency does not always determine the pitch of the sound. The amplitudes of the components of the spectrum are determined by a set of functions known as Bessel functions and represented as $B_n(i)$. An in-depth mathematical study of Bessel functions is beyond the scope of this course; we will only introduce the basics in order to understand how they determine the amplitude of the partials of an FM-generated sound. The figure below shows the graphical representation of four Bessel functions: $B_0(i)$, $\boldsymbol{B}_1(i)$, $\boldsymbol{B}_2(i)$ and $B_3(i)$, respectively. They determine amplitude scaling factors for pairs of sidebands, according to their position relative to the offset carrier's frequency. Note that these are not absolute amplitude values, but scaling factors. The carrier amplitude (a_c) usually defines the overall loudness of the sound, but the amplitudes for individual partials are calculated by scaling the given carrier amplitude according to the factors established by the Bessel functions.

Synthesising time-varing spectra

The ability to provide control for time-varying spectral components of a sound is of critical importance for sound synthesis. The amplitudes of the partials produced by most acoustic instruments vary through their duration. They often evolve in complicated ways, particularly during the attack of the sound. This temporal evolution of the spectrum cannot be heard explicitly at all times. Occasionally, the evolution might occur over a very short time span or the whole duration of the sound itself may be very short. Even so, it establishes an important cue for the recognition of timbre. Frequency modulation offers an effective parameter for spectral evolution: the modulation index (i). As has been already demonstrated, the modulation index defines the number of partials in the spectrum. An envelope can thus be employed to time-vary the modulation index in order to produce interesting spectral envelopes that are unique to FM. Note, however, that by linearly increasing the modulation index the instrument does not necessarily increase the power of high-order sidebands linearly. Remember that the evolution of each partial is determined by its corresponding Bessel function. A specific partial may therefore increase or decrease its amplitude, according to the slope of its Bessel function at specific modulation values.



Frequency ratios and sound design

Timbre control in FM is governed by two simple ratios between FM parameters. One is the ratio between the frequency deviation and the modulator frequency and has already been introduced: it defines the modulation index (*i*). The other is the ratio between the offset carrier frequency and the modulator frequency, called frequency ratio and represented as f_c:f_m. The frequency ratio is a useful tool for the implementation of a phenomenon that is very common among conventional instruments, that is, achieving variations in pitch whilst maintaining the timbre virtually unchanged. If the frequency ratio and the modulation index of a simple FM instrument are maintained constant but the offset carrier frequency is modified then the sounds will vary in pitch, but their timbre will remain unchanged. In this case, it is much easier to think in terms of frequency ratios rather than in terms of values for f_c and f_m separately. For example, whilst it is clear to see that f_c = 220 Hz and f_m = 440 Hz are in ratio 1:2, when presented with the figures f_c = 465.96 Hz and f_m = 931.92 Hz, it is not so obvious. As a rule of thumb, frequency ratios should always be reduced to their simplest form. For example, 4:2, 3:1.5 and 15:7.5 are all equivalent to 2:1. Basic directives for sound design in terms of these simpler ratios are given as follows:

- Case 1: if f_c is equal to any integer and f_m is equal to 1, 2, 3 or 4, then the resulting timbre will have a distinctive pitch, because the offset carrier frequency will always be prominent.
- Case 2: if f_c is equal to any integer and f_m is equal to any integer higher than 4, then the modulation produces harmonic partials but the fundamental may not be prominent.
- Case 3: if f_c is equal to any integer and f_m is equal to 1, then the modulation produces a spectrum composed of harmonic partials; e.g. the ratio 1:1 produces a sawtooth-like wave.
- Case 4: if f_c is equal to any integer and f_m is equal to any even number, then the modulation produces a spectrum with some combination of odd harmonic partials; e.g. the ratio 2:1 produces a square-like wave.
- Case 5: if f_c is equal to any integer and f_m is equal to 3, then every third harmonic partial of the spectrum will be missing; e.g. the ratio 3:1 produces narrow pulse-like waves.
- Case 6: if f_c is equal to any integer and f_m is not equal to an integer, then the modulation produces non-harmonic partials; e.g. 2:1.29 produces a "metallic" bell sound

Sound Examples

Example 1 [Click to hear]
Example 2 [Click to hear]
Example 3 [Click to hear]

form.